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CYCLICAL IMPACTS ON THE PERSONAL DISTRIBUTION OF INCOME

BY CHARLES M. BEACH*

This paper advances a new method for analyzing short-run cyclical changes in inequality in the size distribution of income. Instead of fitting probability densities or studying aggregate measures of inequality such as a Gini coefficient, this distribution-free approach focusses on fluctuations in a set of disaggregative income quantiles spanning a distribution. It is applied to seven age groups of adult male income recipients, and a systematic pattern of cyclical fluctuation in income concentration is revealed, particularly so at the lower end of the distributions for prime aged males.

I. INTRODUCTION

With the initiation of the war on poverty and the growth of public interest in the "quality of life", attention has been turning toward problems of the distribution of output. One aspect of this shift in concerns is the recent interest in the distributional characteristics of macroeconomic fluctuations. Unemployment is viewed, for example by Phelps [19], not just as an indicator that aggregate output is failing to grow at its potential, but also as an economic burden that is borne unequally by members of the population. Associated with different points on a Phillips curve trade-off are different distributions of income. To the extent that monetary and fiscal policy affect aggregate output and employment, they can also affect the distribution of that output. This paper attempts to analyze the distributional aspects of cyclical changes in aggregate economic activity, and thus to serve as a basis for an analysis of the distributional aspects of government stabilization policy.

Johnson [10], Tobin [27], and Hollister and Palmer [9] have argued the importance of low unemployment policies largely on distributional grounds. And Mirer [17], Schultz [21], Thurow [26], and Metcalf [14] have attempted to measure the impacts of macroeconomic fluctuations upon inequality in the size distribution of income. This paper is written in the spirit of the latter contributions, but offers an approach that differ substantially from the earlier ones. Whilst Schultz focused on aggregate inequality measures such as the Gini coefficient, this paper examines inequality behaviour at a disaggregative level within distributions. Metcalf and Thurow based their studies on fitting lognormal and incomplete beta functions to empirical income distributions; but the present approach is distribution-free in that it does not constrain the data to satisfy any particular distribution. And unlike Mirer's microsimulation approach, the current procedure analyzes Bureau of the Census time series distribution data.

The approach used in this study involves characterizing a distribution by a set of quantile income levels and relating short-run fluctuations in each of these quantiles to corresponding fluctuations in macroeconomic activity via a set of

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constrained reduced-form equations. Measures of income inequality are then expressed in terms of these quantiles, and the implied cyclical behaviour of income inequality is derived from the estimated quantile reduced-form equations. Fluctuations in economic aggregates thus affect inequality only indirectly via their impacts on a set of income quantiles. Such an "indirect quantile approach" makes more efficient use of the distributional data available than previous studies, while offering substantially more flexibility and a high level of disaggregation. The approach is quite general and can be applied to many different situations, but is illustrated in this paper with a set of distributions of male income recipients disaggregated by age.

The outline of the paper is as follows. The next two sections set out and implement the basic model of income changes within a distribution. Section IV discusses the estimation procedure and results based on annual Bureau of the Census data. Sections V and VI then examine the implied partial and total behaviour of cyclical fluctuations in income concentration. Some possible extensions of the basic model and approach are then discussed in the concluding Section VII.

II. A Model of Quantile Income Changes

The basic idea behind the indirect quantile approach is to link changes in key macroeconomic variables to changes in the pattern of income concentration via their effects on a set of quantile income levels. Consequently, the first stage in the analysis is the development of a simple reduced-form model of the channels through which macro activity affects individual quantile income levels. The second stage, expressing measures of income concentration in terms of these income quantiles, will be developed below in Section V.

In the analysis that follows the basic income-receiving unit will be the individual rather than the family. This avoids complications associated with changing composition of family units,[1] and with cyclical fluctuations in the number of family units.[2] Also, as an empirical convenience, the quantiles selected for study will be the nine income deciles $y(1), y(2), \ldots, y(9)$,[3] although a finer set of quantiles could be readily employed.[4]

Now $y(i)$ can be decomposed according to a stochastic identity into income components derived from different sources:

$$(1) \qquad y(i) = YE(i) + YU(i) + YPF(i) + YTR(i) + YTP(i) + YK(i) + v(i)$$

[1] See [21], p. 73.

[2] See [22], pp. 512–513.

[3] The i-th decile represents that income point such that $10i$ percent of the distribution have incomes less than or equal to this level and $100-10i$ percent have incomes that exceed it. We are modelling the behaviour of relative income positions and not the income levels of particular individuals. Relative income groups are thus similar to but not the same as a constant group of individuals. Particular occupational groups, for example, might move from one relative income group to another over time, so that the mean for a particular relative income group pertains to a shifting group of individuals.

[4] See, for example, Gray [8] in which a comparison is made of black and white income distributions in terms of vigintiles.

where $YE(i)$ is the average income received from employment; $YU(i)$ is average unemployment benefits received; $YPF(i)$ represents farm proprietary income; $YTR(i)$ and $YTP(i)$ are relief transfers and pension transfers; $YK(i)$ is average capital income in the form of rents, interest, and dividends; and $v(i)$ is a random term assumed to represent remaining minor sources of income. The first two labour income components can be further factored into

$$YE(i) = PR(i) \cdot ER(i) \cdot W(i)$$

and

(2)
$$YU(i) = PR(i) \cdot UR(i) \cdot UB(i),$$

where $PR(i)$ is the quantile group's average labour force participation rate; $ER(i) = 1 - UR(i)$ is its employment rate; $W(i)$ is wage and salary income per employed person in the group; and $UB(i)$ is the average unemployment benefits per unemployed person. Content is now given to these relationships by (a) relating the decile-specific variables in these equations to economic aggregates, and (b) selecting which variables in (1) and (2) to retain or drop for particular decile groups.

Since the right-hand side variables in (1) and (2) are generally unobserved, they will be assumed to be nondecreasing linear functions of corresponding observed economic aggregates. That is,

(3)
$$\begin{aligned}
PR(i) &= \delta_{PR}(i) + \alpha_{PR}(i)PR & \alpha_{PR}(i) &\geq 0 \\
ER(i) &= \delta_{ER}(i) + \alpha_{ER}(i)ER & \alpha_{ER}(i) &\geq 0 \\
W(i) &= \delta_{W}(i) + \alpha_{W}(i)W & \alpha_{W}(i) &\geq 0 \\
UB(i) &= \delta_{UB}(i) + \alpha_{UB}(i)W & \alpha_{UB}(i) &\geq 0 \\
YPF(i) &= \delta_{PF}(i) + \alpha_{PF}(i)YPF & \alpha_{PF}(i) &\geq 0 \\
YTR(i) &= \delta_{TR}(i) + \alpha_{TR}(i)YTR & \alpha_{TR}(i) &\geq 0 \\
YTP(i) &= \delta_{TP}(i) + \alpha_{TP}(i)YTP & \alpha_{TP}(i) &\geq 0 \\
YK(i) &= \delta_{K}(i) + \alpha_{K}(i)YK & \alpha_{K}(i) &\geq 0
\end{aligned}$$

It will be noted that unemployment benefits have been assumed a function of average wage income, W, because of the institutional fashion in which individuals' unemployment benefits are closely tied to their recent wage earnings.

When the relationships in (2) and (3) are substituted into (1), the resulting equations can be written compactly for all n observations as

(4)
$$y(i) = x\beta(i) + v(i)$$

where $y(i)$ is now a column vector of observations on the i-th decile; x is an $n \times 12$ matrix of observations on terms[5] involving the right-hand side variables in (3); $\beta(i)$ is a conformable column vector of coefficients derived from the alpha and delta coefficients in (3); and $v(i)$ is now an n-dimensional vector of random terms. In general, the elements of x include observations on composite terms such as

[5] The twelve variables in x being ER, PR, $PR \cdot ER$, $ER \cdot W$, $PR \cdot W$, W, $PR \cdot ER \cdot W$, YPF, YTR, YTP, YK, and the constant term.

$PR \cdot W$ or $PR \cdot ER$ as well as on simple terms such as YPF and YTR. Consequently, (4) is linear in the betas, but nonlinear in the aggregate variables of the analysis. The betas are also, in general, nonlinear functions of the coefficients in (3). The non-negativity constraints on the alphas imply that the coefficients corresponding to the simple terms YPF, YTR, YTP, and YK are also non-negative. Expressing the decile incomes in the form of (4), one can then characterize changes in the overall income distribution for each group by the set of nine decile equations,

$$\begin{bmatrix} y(1) \\ \vdots \\ y(9) \\ \end{bmatrix} = \begin{bmatrix} x & & & 0 \\ & \cdot & & \\ & & \cdot & \\ & & & \cdot \\ 0 & & & x \end{bmatrix} \begin{bmatrix} \beta(1) \\ \vdots \\ \beta(9) \\ \end{bmatrix} + \begin{bmatrix} v(1) \\ \vdots \\ v(9) \\ \end{bmatrix}$$

or more compactly,

$$y = X\beta + v. \tag{5}$$

In summary, then, equation (5) with its appropriately signed coefficients sets out in formalistic fashion the channels through which decile changes occur in the income distribution.

III. Implementation and Model Constraints

To allow for a more detailed comparative analysis of short-run distributional behaviour, the above model has been implemented for seven different age groups of adult males: ages 14–19, 20–24, 25–34, 35–44, 45–54, 55–64, and 65 and over.

Now we should not be very optimistic about estimating (5) with a paucity of observations since it contains twelve coefficients for each decile equation. Consequently a second set of *a priori* constraints specifying the beta coefficients on some of the labour income terms to be zero has been imposed. In particular, the labour income terms in (4) have been assumed to fall into four categories. In Case I (assumed applicable for low-income prime-aged males), changes in both participation and employment rates are assumed to have proportional impacts on decile incomes[6] so that the labour income terms in (4) simplify to

$$\beta_3(i)PR \cdot ER + \beta_5(i)PR \cdot W + \beta_7(i)PR \cdot ER \cdot W. \tag{6}$$

In Case II, everyone in the labour force in the decile group is assumed to be employed, but not everyone is a full-time participant in the labour force.[7] This situation is assumed to characterize the upper-income groups sixty-five and over who would be expected to retire from the labour force should they become unemployed; the upper-income groups aged twenty to twenty-four who would not be expected to suffer much unemployment, but some of whose members may not be in the labour force the full year because of educational commitments; and

[6] ie: $\delta_{PR} = \delta_{ER} = \delta_{UB} = 0$.
[7] ie: $\delta_{PR} = \alpha_{ER} = 0$, $\delta_{ER} = 1$.

also the bottom deciles for the young group fourteen to nineteen whose incomes are so low[8] that their members are assumed to be only marginally attached to the labour force.[9] In Case III (assumed relevant for some older middle-income groups), all members of a decile group are assumed to be fully employed throughout the year.[10] And in Case IV (basically for middle- and upper-income prime-aged males), all members of a particular decile group are assumed to be fully participating in the labour force and fully employed.[11] The bottom two deciles for the oldest group sixty-five and over were, in addition, taken to be independent of wage fluctuations since their labour force attachment is assumed to be minimal.[12]

The nonlabour terms also need to be further constrained. Farm proprietary income and relief transfers will occur in equations only for low decile income levels among recipients aged twenty to sixty-four. Pension benefits are assumed to be received only by those of retirement age. Capital income accrues in substantial proportions only to the very top quantile groups among prime aged males and to those sixty-five and over.[13] Since the top quantile estimated in the model (the 90th percentile) lies substantially below this high income group, capital income has been omitted from all decile groups but those sixty-five and over. Thus only a fraction of the terms in equation (4) will appear in the equation for any particular decile level, and the matrix of independent variables in (5) may be rewritten as

$$X = \begin{bmatrix} x(1) & & 0 \\ & \ddots & \\ 0 & & x(9) \end{bmatrix}$$

where now the submatrices of independent variables corresponding to each decile equation are no longer the same.

Finally, there exists a set of adding-up constraints upon the coefficients in (5). Basically we have been trying to allocate the changes in aggregate independent variables among a set of individual income levels. But in so doing, one would wish that the individual impacts would sum up to the total impact of the initial change—or alternatively, the average impact upon the individual income quantiles would equal the mean impact upon the distribution; i.e.,

$$(7) \qquad \int_i \left(\frac{\partial y(i) f(y)}{\partial W}\right) dy(i) = \frac{\partial \mu}{\partial W}$$

[8] The four lowest income deciles had estimated mean values of \$89.70, \$179.60, \$269.60, and \$359.50 over the 1947–1970 period.

[9] This is not to deny a potentially powerful *indirect* effect of employment rates upon income via participation rates according to discouraged worker or additional worker effects. See also the discussion at the end of Section V below.

[10] ie: $\delta_{ER} = \delta_{UB} = \alpha_{PR} = 0$, $\delta_{PR} = 1$.

[11] ie: $\delta_{PR} = \delta_{ER} = 1$, $\alpha_{PR} = \alpha_{ER} = 0$.

[12] In [20], Table 1 it is estimated that groups aged sixty-five and over with family income under \$3,000 in 1962 (ie: the 47th percentile) received only 14 percent of their income in the form of wages and salaries, 11 percent in property income, and 74 percent in pensions, annuities, and other income.

[13] In *op. cit.*, it is shown that for groups aged less than sixty-five, only those with family incomes above \$25,000 in 1962 (ie: essentially the top 1 percentile) received on average more than 9 percent of their incomes from capital.

where $f(y)$ is the probability density function of an age-specific income distribution and μ is the corresponding mean of the distribution. Analogous constraints can also be formulated for each of the other independent variables in the model. But since only nine quantile equations are estimated for each distribution, (7) must be implemented as an approximation discussed in Appendix I of this paper. As there indicated, these adding-up constraints on the beta coefficients across decile equations can in general be specified in the conventional linear form

$$R\beta = r \tag{8}$$

where R and r have number of rows equal to the number of adding-up constraints imposed on the coefficients of the quantile equations.

In summary, then, the "seemingly unrelated" equations in (5) together with the three sets of constraints introduced above constitute the proposed model of decile income behaviour that is estimated in this paper.

IV. Estimation of the Model

In order to estimate the quantile model subject to the adding-up constraints in (8), estimates were first obtained for the dependent income decile levels by linear interpolation from the Bureau of the Census Series P-60 on *Consumer Income* for each year from 1947 to 1970. Since these estimates of the dependent variables are assumed to be related to the true decile levels by an additive error term, the equation in (5) can be rewritten with a new error term,

$$y = X\beta + u, \tag{9}$$

where the intercepts have been adjusted so that u has zero mean.

Data on the independent variables in (9) have been drawn from the annual national accounts in the *Survey of Current Business* (and deflated by adult population figures drawn from the Census Bureau's Series P-25 on *Population Estimates*) and from the *Handbook of Labor Statistics*. Thus YTR, for example, is mean relief transfers per adult in the United States, while YTP is mean old-age benefits and pension transfers per person aged sixty-five and over, W is mean wage and salary income per employed adult male in the population, and PR and ER are age-specific male participation and employment rates.

Since the adding-up constraints employed in the regression analysis are only approximations to true constraints in (7), they have been imposed upon the coefficients of only those variables that appear in almost all the nine decile equations for a given age group. Consequently the constraints actually imposed are those corresponding to the wage income and participation rate variables for the first, second, and seventh age groups, and to W alone for the remaining four prime-age groups.

Several econometric problems now arise in estimating the model specified in (8) and (9). First of all, inspection of (6) reveals that, in the Case I situation, two of the labour income terms (with coefficients β_5 and β_7) are extremely collinear. The approach taken to handle this multicollinearity problem is to specify the $\beta_5(i)$ coefficients *a priori*, and then fit the equations subject to these additional restrictions. This procedure is facilitated by the fact that β_5 is simply $\alpha_{PR}(i) \cdot \alpha_{UB}(i)$.

where $\alpha_{UB} = UB(i)/W$ and $\alpha_{PR} = PR(i)/PR$. Thus, given observations on PR and W, it is sufficient to specify values for only $UB(i)$, the unemployment benefits members of a particular decile group receive on average, and $PR(i)$, their average participation rate. Thus resulting *a priori* beta coefficients are shown in Table I below with no "t-ratios".

Secondly, since the study uses time series data, it would be expected that the disturbances in (9) would be serially interdependent as well as contemporaneously correlated. Parks [18] and Kmenta and Gilbert [11] suggested a procedure for handling first-order serial correlation in the framework of Zellner's seemingly unrelated equations without cross-equation coefficients constraints. The estimators used in this study generalize Parks' procedure so as also to incorporate the adding-up constraints. For details on the resulting "constrained Parks" estimation procedure and proof of its conventional asymptotic properties, see Appendix II.

The constrained Parks coefficients estimates together with their "t-ratios" are set out in Table 1 with each block of results corresponding to a different age group. The dependent variables for each set of equations are listed across the top of each block, and the terms appearing on the right-hand side of each equation are listed down the left-hand column. Thus by glancing down a column, one can read off the coefficient estimates for any equation within a given block. All the coefficients with *a priori* expected signs turn out to have their correct signs. The R^2's (although not entirely appropriate in a multi-equation framework) vary from 0.48 to 0.82 over the first nine equations, and then rise to 0.98 or above in forty-nine of the remaining fifty-four equations. The estimated autocorrelation coefficients have t-rations of 2.0 or more in twenty-six of the sixty-three equations.

Of more interest for the purposes of this paper, however, are the distributional impact patterns implicit in the above regression results. Table 2, laid out in the same format as Table 1 contains the partial elasticities (evaluated at the mean) of the decile income levels estimated from the above equations. According to the model developed in the previous section, all of these elasticities should be non-negative; and as can be seen in Table 2, all but one are.

To assist in interpreting these figures, several highlights should perhaps be commented on. Firstly, one may notice a rather substantial difference in the impact patterns between the secondary age groups, fourteen to nineteen and sixty-five and over, and the remaining groups aged twenty to sixty-four. Fluctuations in participation and employment rates for the latter group have their greatest impacts at the lower end of the income distribution; while, for the former secondary aged group, the strongest impacts occur at the middle and upper portions of the distributions since the bottom decile groups likely have the weakest labour force attachment among adult males. Similarly, most males twenty-five to sixty-four are fully employed and receiving incomes almost entirely in the form of wages and salaries; so it should not be surprising that their wage income elasticities are approximately unity. Among younger and older workers, however, the wage income elasticities differ noticeably from unity, particularly among the lower deciles. Among older workers, this may be due to most low-income males sixty-five and over receiving their incomes largely from

TABLE 1

CONSTRAINED PARKS ESTIMATES OF THE REGRESSION COEFFICIENTS

Ages 14–19

	y(1)	y(2)	y(3)	y(4)	y(5)	y(6)	y(7)	y(8)	y(9)
Constant	54.68 (1.687)	79.82 (1.515)	103.7 (1.396)	128.5 (1.333)	246.0 (3.836)	154.9 (1.284)	−71.66 (0.422)	−134.4 (0.386)	−951.8 (1.801)
PR	−9.177 (0.201)	31.69 (0.425)	75.01 (0.714)	117.2 (0.860)					
PR · W	0.0099 (5.776)	0.0193 (6.984)	0.0286 (7.418)	0.0379 (7.617)			0.124	0.136	0.148
PR · ER					−127.7 (1.172)	32.63 (0.165)	685.6 (2.565)	803.6 (1.446)	2590.0 (3.139)
PR · ER · W					0.0803 (9.947)	0.1190 (8.697)	0.0114 (0.745)	0.0999 (3.030)	0.2744 (6.018)

Ages 20–24

	y(1)	y(2)	y(3)	y(4)	y(5)	y(6)	y(7)	y(8)	y(9)
Constant	−947.9 (2.606)	−1301.0 (3.174)	−826.3 (1.563)	78.08 (1.342)	534.3 (1.145)	1248.0 (2.640)	1692.0 (1.675)	3273.0 (4.518)	3097.0 (3.285)
PR · ER	1715.0 (3.651)	2355.0 (4.641)	1882.0 (2.921)	653.0 (0.934)	168.8 (0.305)	−651.3 (1.160)			
PR · ER · W	−0.1157 (7.059)	−0.1349 (7.966)	−0.1232 (5.621)	0.0192 (1.214)	0.0989 (7.589)	0.1950 (12.39)			
PR							−1025.0 (0.940)	−2717.0 (3.467)	−2425.0 (2.371)
PR · W	0.095	0.153	0.239	0.253	0.267	0.278	0.5566 (34.00)	0.6664 (55.25)	0.8156 (62.01)
YTR	11.53 (3.827)	14.42 (4.796)	11.38 (3.483)						
YPF	0.5862 (0.972)	0.7134 (1.247)							

TABLE 1 (Continued)

Ages 25–34

	y(1)	y(2)	y(3)	y(4)	y(5)	y(6)	y(7)	y(8)	y(9)
Constant	−5204.0 (3.987)	−3306.0 (1.990)	−1126.0 (1.663)	742.5 (1.584)	440.6 (17.01)	438.4 (18.20)	393.5 (12.49)	−35.52 (0.255)	−74.32 (0.510)
PR · ER	5348.0 (3.726)	3623.0 (1.974)	1586.0 (2.176)	−332.9 (0.657)					
PR · ER · W	0.1230 (6.732)	0.2365 (15.92)	0.3028 (52.20)	0.3915 (84.95)					
PR · W	0.175	0.231	0.275	0.280					
W					0.7262 (183.0)	0.8157 (219.6)	0.9231 (193.4)	1.133 (57.77)	1.371 (66.90)
YTR	11.41 (3.080)								
YPF	1.968 (2.573)	0.8503 (0.529)							

Ages 35–44

	y(1)	y(2)	y(3)	y(4)	y(5)	y(6)	y(7)	y(8)	y(9)
Constant	−6737.0 (4.318)	−2706.0 (2.405)	369.3 (0.513)	345.4 (4.978)	389.2 (5.024)	85.69 (1.167)	−91.66 (0.683)	−109.9 (0.525)	−756.5 (2.045)
PR · ER	6678.0 (3.927)	2794.0 (2.300)	−164.0 (0.215)						
PR · ER · W	0.2022 (11.71)	0.3338 (36.54)	0.3956 (52.00)						
PR · W	0.174	0.230	0.279						
W				0.7341 (72.75)	0.8281 (76.30)	0.9848 (92.58)	1.148 (61.53)	1.334 (44.98)	1.833 (34.56)
YTR	9.087 (2.783)								
YPF	1.980 (2.180)	0.8502 (1.285)							

TABLE 1 (Continued)

Ages 45–54

	$y(1)$	$y(2)$	$y(3)$	$y(4)$	$y(5)$	$y(6)$	$y(7)$	$y(8)$	$y(9)$
Constant	−8.747 (5.424)	−5586.0 (3.936)	−1793.0 (1.519)	−313.8 (0.303)	2377.0 (2.282)	68.67 (0.883)	−252.8 (1.333)	−465.1 (2.479)	−1253.0 (2.774)
$PR \cdot ER$	8514.0 (4.784)	5432.0 (3.451)	1633.0 (1.283)						
$PR \cdot ER \cdot W$	0.1277 (6.199)	0.3132 (16.60)	0.4265 (42.22)						
$PR \cdot W$	0.178	0.216	0.275						
ER				384.2 (0.354)	−2299.0 (2.108)				
$ER \cdot W$				0.4632 (59.70)	0.5580 (67.13)				
W				0.275	0.275	0.9414 (84.00)	1.133 (41.98)	1.365 (51.02)	1.936 (29.65)
YTR	25.24 (6.044)	12.88 (3.329)							
YPF	2.841 (3.293)	1.432 (1.779)							

TABLE 1 (Continued)

Ages 55–64

	y(1)	y(2)	y(3)	y(4)	y(5)	y(6)	y(7)	y(8)	y(9)
Constant	−1831.0 (2.592)	−1341.0 (1.053)	−1536.0 (1.384)	165.3 (0.203)	2353.0 (2.440)	168.3 (1.611)	113.7 (0.861)	−296.1 (1.030)	−350.8 (0.745)
PR · ER	1364.0 (1.583)	859.4 (0.550)	1293.0 (0.969)	−463.1 (0.485)					
PR · ER · W	0.1038 (7.007)	0.2292 (8.683)	0.3252 (16.30)	0.4371 (24.56)					
PR					−2797.0 (2.625)				
PR · W	0.137	0.185	0.260	0.301	0.8336 (46.90)				
W						0.7952 (52.89)	0.9326 (48.71)	1.199 (28.91)	1.580 (24.89)
YTR	5.148 (1.958)	8.820 (1.964)	6.415 (2.029)						
YPF	2.974 (4.998)	1.960 (1.933)	1.033 (1.459)						

Ages 65+

	y(1)	y(2)	y(3)	y(4)	y(5)	y(6)	y(7)	y(8)	y(9)
Constant	12.02 (0.450)	182.7 (7.578)	185.0 (1.810)	−54.01 (0.443)	67.95 (0.444)	−193.0 (0.958)	699.3 (3.527)	624.3 (1.384)	1634.0 (2.154)
YTP	0.4893 (24.43)	0.5938 (32.27)	0.6904 (13.75)	0.9130 (15.08)	0.9786 (13.56)	0.8714 (6.960)	0.3312 (1.920)		
PR · ER			140.4 (0.552)	908.9 (2.954)					
PR · ER · W			0.0175 (0.351)	−0.0214 (0.353)					
PR					652.2 (1.868)	1508.0 (3.357)	240.0 (0.605)	285.1 (0.279)	−1758.0 (1.134)
PR · W			0.028	0.063	0.1416 (1.958)	0.2549 (2.772)	0.3526 (3.890)	0.8144 (3.473)	1.355 (3.621)
YK						1.446 (3.117)	4.417 (4.596)	6.823 (5.996)	9.051 (5.212)

TABLE 2

ESTIMATED DECILE INCOME ELASTICITIES

Ages 14–19

	y(1)	y(2)	y(3)	y(4)	y(5)	y(6)	y(7)	y(8)	y(9)
PR	0.364	0.532	0.593	0.620	0.437	0.707	1.07	1.10	1.48
ER	0	0	0	0	0.437	0.707	0.499	0.658	1.20
W	0.429	0.418	0.414	0.411	0.592	0.676	0.615	0.723	0.755

Ages 20–24

	y(1)	y(2)	y(3)	y(4)	y(5)	y(6)	y(7)	y(8)	y(9)
PR	2.25	1.96	1.43	0.962	0.795	0.608	0.553	0.264	0.412
ER	1.40	1.18	0.601	0.305	0.243	0.137	0	0	0
W	−0.107	0.143	0.404	0.703	0.742	0.799	0.796	0.813	0.810
YTR	0.273	0.191	0.104	0	0	0	0	0	0
YPF	0.107	0.073	0	0	0	0	0	0	0

Ages 25–34

	y(1)	y(2)	y(3)	y(4)	y(5)	y(6)	y(7)	y(8)	y(9)
PR	3.50	2.05	1.30	0.827	0	0	0	0	0
ER	2.92	1.58	0.866	0.445	0	0	0	0	0
W	0.882	0.917	0.902	0.901	0.909	0.918	0.933	1.00	1.01
YTR	0.083	0	0	0	0	0	0	0	0
YPF	0.101	0.031	0	0	0	0	0	0	0

Ages 35–44

	y(1)	y(2)	y(3)	y(4)	y(5)	y(6)	y(7)	y(8)	y(9)
PR	4.00	1.79	0.910	0	0	0	0	0	0
ER	3.52	1.38	0.511	0	0	0	0	0	0
W	1.03	0.989	0.948	0.927	0.927	0.982	1.01	1.01	1.07
YTR	0.060	0	0	0	0	0	0	0	0
YPF	0.102	0.028	0	0	0	0	0	0	0

TABLE 2 (Continued)

Ages 45–54

	y(1)	y(2)	y(3)	y(4)	y(5)	y(6)	y(7)	y(8)	y(9)
PR	6.08	2.86	1.47	0	0	0	0	0	0
ER	5.43	2.41	1.05	0.693	0.204	0	0	0	0
W	1.10	1.07	1.07	0.983	0.968	0.985	1.03	1.05	1.11
YTR	0.225	0.065	0	0	0	0	0	0	0
YPF	0.196	0.056	0	0	0	0	0	0	0

Ages 55–64

	y(1)	y(2)	y(3)	y(4)	y(5)	y(6)	y(7)	y(8)	y(9)
PR	2.51	1.54	1.48	0.952	0.453	0	0	0	0
ER	1.75	1.01	0.988	0.510	0	0	0	0	0
W	1.31	1.15	1.09	1.06	1.02	0.962	0.976	1.03	1.03
YTR	0.077	0.067	0.033	0	0	0	0	0	0
YPF	0.342	0.116	0.041	0	0	0	0	0	0

Ages 65 & Over

	y(1)	y(2)	y(3)	y(4)	y(5)	y(6)	y(7)	y(8)	y(9)
PR	0	0	0.133	0.297	0.321	0.517	0.313	0.508	0.422
ER	0	0	0.077	0.120	0	0	0	0	0
W	0	0	0.089	0.067	0.182	0.261	0.281	0.481	0.537
YTP	0.978	0.785	0.704	0.748	0.650	0.461	0.137	0	0
YK	0	0	0	0	0	0.131	0.311	0.355	0.316

pensions and old-age assistance. And among younger males wage increases may be dampened by the cyclical entry and exit of young workers particularly at the bottom end of the distributions, and perhaps also by the relatively large post-war increase in the supply of young workers.

Second, is the result that for prime aged males there appears to be a slight U-shape in the wage income elasticities across deciles (as illustrated in Figure 1 for

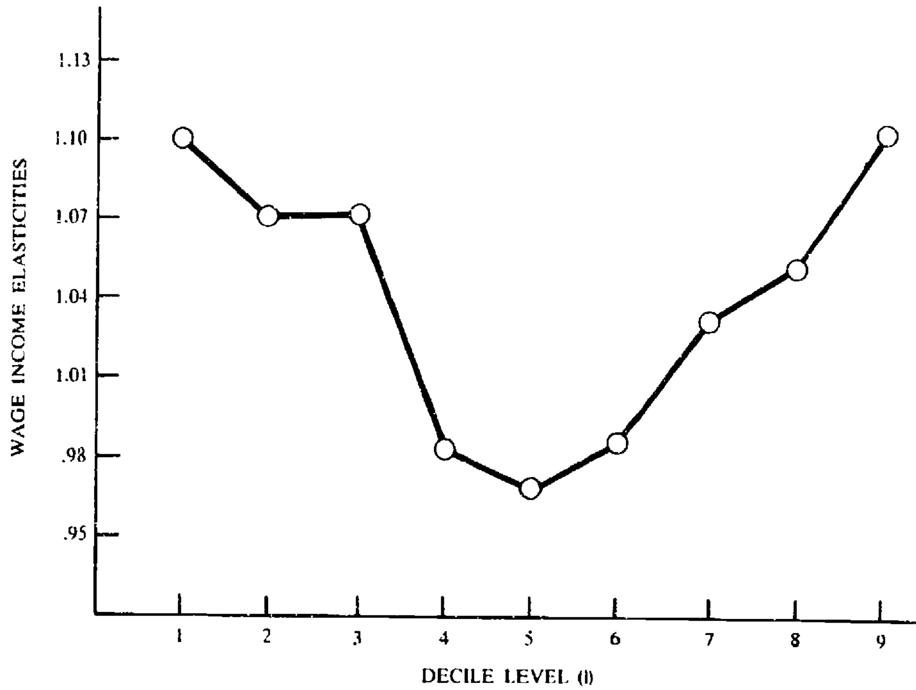


Figure 1 Profile of wage—income elasticities for group aged 45–54

the age group 45–54). One possible hypothesis to account for the dip in the elasticities over the middle decile groups may be the rigidifying effects that union agreements have upon the wage incomes their members receive. Lewis [12] among others has found evidence to suggest that unions have tended to make their members' money wages somewhat rigid against short-run movements in the incomes of non-unionized workers. A recent study by Massad [13] indicates that the effects of unionism are concentrated in the middle deciles of the income distribution for adult males. Consequently, a slight dip in the elasticities around the middle of the prime aged distributions would appear quite reasonable.

Third, non-labour income appears in general to have relatively inelastic effects upon the decile income levels. Relief transfers and farm proprietary income both affect low-income deciles quite inelastically. Interest income has moderate effect on the upper segments of the distribution for older males. And one can also see quite clearly how the impact of pension assistance transfers declines across deciles for the oldest age group.

In summary, then, the estimation results appear on the whole to be quite reasonable and to indicate a substantial difference in the impact patterns between middle-aged males and younger and older male income recipients.

V. Cyclical Sensitivity of Income Inequality

Since the objective of this paper is to analyze cyclical fluctuations in income concentration, attention must now be turned to the use of the above results in an analysis of inequality. In a classic 1945 paper [5] May Jean Bowman argued that no aggregate statistic is entirely satisfactory as an index of inequality. Accordingly, this paper characterizes inequality in a disaggregative fashion by a set of statistics that can be computed from the results already obtained. The indirect quantile approach allows one to examine almost any measure of inequality one might be interested in, but perhaps the simplest to analyze on the basis of our results is a set of relative median income figures, $y(i)/y(5)$, for each distribution. Since we have already obtained estimates of the elasticities of the decile *levels* to fluctuations in key economic aggregates, we can easily obtain the elasticities of the decile *ratios* from the figures in Table 2,[14] and these appear in Table 3. The format of this table differs from that of earlier tables in that each block of figures corresponds to a single independent variable such as wage income or an employment rate. One can thus see the pattern of elasticities over different age groups (by glancing down the columns). It should also be noted that comparison has been eased by transforming age-specific participation and employment rate elasticities in Table 2 into elasticities with respect to average participation and employment rates for *all* adult males.[15]

Several features of the results in Table 3 deserve comment. First of all, it can be seen that (contrary to Schultz' aggregative findings in [21]) income inequality does appear on the whole to be noticeably sensitive to fluctuations in overall participation and employment rates, and less so to cyclical changes in the level of per capita wage income. Indeed at the lower ends of the prime-age distributions, the participation and employment rate impacts turn out to be rather highly elastic.

Second, and as a corollary to the findings of the previous section, the inequality impact patterns presented in Table 3 differ substantially between the youngest and oldest distributions and the rest of the distributions. Postive values *below* the median represent a reduction in income inequality since the corresponding relative median income ratios shift up toward unity, while positive values *above* the median represent an increase in concentration since the corresponding upper deciles are diverging further away from the median of the distribution. Negative entries in Table 3 can be interpreted in converse fashion. Consequently, one can see from the table that, for males twenty to sixty-four, cyclical increase in participation and employment rates result in reduced income inequality (basically

[14] By the simple rule that the elasticity of a ratio is equal to the difference in the elasticities.

[15] The transformation was obtained by using a constrained Parks procedure to regress simultaneously the logarithms of each of the age-specific participation and employment rates on the logarithms of their respective overall average rates subject to the explicit adding-up constraint (thus yielding estimates of the elasticity factors of the age-specific rates with respect to their corresponding overall rates, and then multiplying the age-specific elasticities in Table II by these elasticity factors.

For details, see [1], pp. 149–153.

by raising the lowest deciles up towards the medians), while the effects of wage increases depend upon the age and decile position of a particular group. Among younger workers twenty to thirty-four, cyclical increases in wage income tend to be disequalizing over both upper and lower regions of the distributions. While among older workers thirty-five to sixty-four, incomes tend to become slightly less concentrated over lower portions of the distributions and slightly more so over the upper portions—essentially as a consequence of the slight U-shape in the profile of wage-income elasticities across deciles that was pointed out in the previous section. And finally, among young and older secondary aged males, increases in wage income tend to be generally disequalizing, while participation and employment rate changes have mixed effects.

More generally, then, inequality among secondary aged males fourteen to nineteen and sixty-five and over tends to fluctuate procyclically, while among the

TABLE 3

ESTIMATED RELATIVE MEDIAN INCOME ELASTICITIES ACROSS AGES AND DECILES

(a) Wage Income Elasticities

deciles: / age:	1	2	3	4	5	6	7	8	9
14–19	−0.162	−0.174	−0.178	−0.181	0	0.084	0.023	0.131	0.164
20–24	−0.849	−0.599	−0.338	−0.039	0	0.057	0.054	0.071	0.068
25–34	−0.027	0.008	−0.007	−0.008	0	0.009	0.024	0.094	0.097
35–44	0.102	0.062	0.021	0.001	0	0.056	0.084	0.082	0.142
45–54	0.134	0.100	0.100	0.016	0	0.018	0.067	0.084	0.144
55–64	0.291	0.133	0.074	0.043	0	−0.055	−0.042	0.019	0.012
65+	−0.182	−0.182	−0.094	−0.115	0	0.079	0.099	0.299	0.355

(b) Participation Rate Elasticities

deciles: / age:	1	2	3	4	5	6	7	8	9
14–19	−0.119	0.159	0.259	0.304	0	0.447	1.04	1.09	1.73
20–24	1.40	1.11	0.608	0.160	0	−0.179	−0.232	−0.510	−0.368
25–34	2.37	1.39	0.882	0.560	0	0	0	0	0
35–44	2.88	1.29	0.654	0	0	0	0	0	0
45–54	4.15	1.95	1.00	0	0	0	0	0	0
55–64	1.59	0.841	0.798	0.387	0	−0.351	−0.351	−0.351	−0.351
65+	−1.49	−1.49	−0.872	−0.113	0	0.908	−0.036	0.869	0.466

(c) Employment Rate Elasticities

deciles: / age:	1	2	3	4	5	6	7	8	9
14–19	−0.952	−0.952	−0.952	−0.952	0	0.589	0.137	0.482	1.655
20–24	2.18	1.78	0.678	0.118	0	−0.200	−0.458	−0.458	−0.458
25–34	2.89	1.56	0.858	0.440	0	0	0	0	0
35–44	2.60	1.02	0.377	0	0	0	0	0	0
45–54	3.63	1.53	0.585	0.339	0	−0.142	−0.142	−0.142	−0.142
55–64	1.37	0.794	0.774	0.400	0	0	0	0	0
65+	0	0	0.050	0.127	0	0	0	0	0

remaining groups aged twenty to sixty-four, it tends to move anticyclically at least in so far as the participation and employment rate effects are concerned. This basically reflects the hypothesis that the very low-income secondary workers have only a very weak commitment to the labour force so that it is the middle- and upper-income secondary groups whose incomes are more responsive to participation and employment rate fluctuations. Those who are still employed at age sixty-five and over receive more on average than those who are living off pension and old-age assistance income—thus cyclical upturns tend to raise already relatively high incomes and recessions tend to low them back towards the median of the distribution. Among lower decile teenage income recipients on the other hand, some may have part-time or part-year employment in service jobs that are not as cyclically sensitive as higher paid industrial jobs. In addition, the relatively large increase in the supply of young workers over the post-World War II period may have tended to dampen expansionary wage increases employed members of this group receive.[16] As a further consideration, it could be argued that the procyclical behaviour of income inequality over the lower portions of both distributions may also be partially due to the choice of the individual as the income receiving unit used in this study. In recessions, formerly low-income secondary workers may simply drop out of the labour force, earn no income, and thus not be counted in the income distribution; while in economic expansions, they may re-enter the lower end of the distribution, thus attenuating or concealing any upturn in relative median incomes that might otherwise have occurred. Consequently, one must be particularly careful about making normative statements concerning the behaviour of income inequality of these two distributions.

VI. Combined Cyclical Changes in the Distribution

Cyclical changes in the pattern of income inequality, however, are the result not of macro variables fluctuating singly in isolation but of their simultaneous variation. What we are basically interested in, then, are the *total* or combined effects of fluctuations in such factors as wages, participation rates, and employment (or unemployment) rates rather than just the *partial* effects analyzed in the last section. The model of quantile behaviour so far presented, however, has incorporated no macroeconomic behavioural assumptions, but has attempted simply to translate given changes in key economic aggregates into disaggregated distributional changes. Consequently, such a quantile model can be readily appended to any annual macro model that simultaneously determines these economic aggregates so as to analyze the total distributional impacts of their fluctuations. In this section are described the results of three simulation experiments based on the labour supply models of Bowen and Finegan [4] and Wachter [28].

Two principal factors that will be assumed to generate cyclical distributional changes among adult males are wage income and unemployment rate changes. But these in turn affect levels of participation rates and unemployment benefits. According to the work of Tella [24], Dernberg and Strand [7], and Bowen and

[16] For a more detailed discussion of this effect, see [1], pp. 209–213.

Finegan, the participation rates of secondary aged males appear to vary inversely with the unemployment rate—the so-called "discouraged worker effect", that during recessions many such workers who have difficulty finding employment simply drop out of the labour force, and that when economic conditions improve, they enter it again responding to improved job opportunities. According to Mincer's neoclassical approach [15, 16], however, participation behaviour should reflect the labour–leisure trade-off by showing sensitivity to real wages. More recently, Wachter has attempted to integrate this latter approach with a permanent income hypothesis and a theory of relative wage adjustment. Increases in permanent real wage income are expected to have a "permanent" effect upon participation behaviour of secondary workers according to the outcome of conflicting income and substitution effects. Furthermore, a deviation of current real wage income from permanent wage income has an additional "transitory" effect of uncertain *a priori* sign upon participation behaviour; but once current wages fall back to their permanent wage level, this transitory effect ceases to operate. Empirically, Wachter finds that for young secondary aged males, the permanent wage effect is negative while the transitory wage effect is not statistically significant; and for older secondary aged males, the permanent effect remains negative, while the transitory effect turns out positive and strong. In summary, then, cyclical fluctuations in economic activity would be expected to affect participation behaviour through three distinct channels: an unemployment rate effect, a permanent wage effect, and a transitory wage effect. The outcome of these sometimes conflicting effects depends on their relative magnitudes.

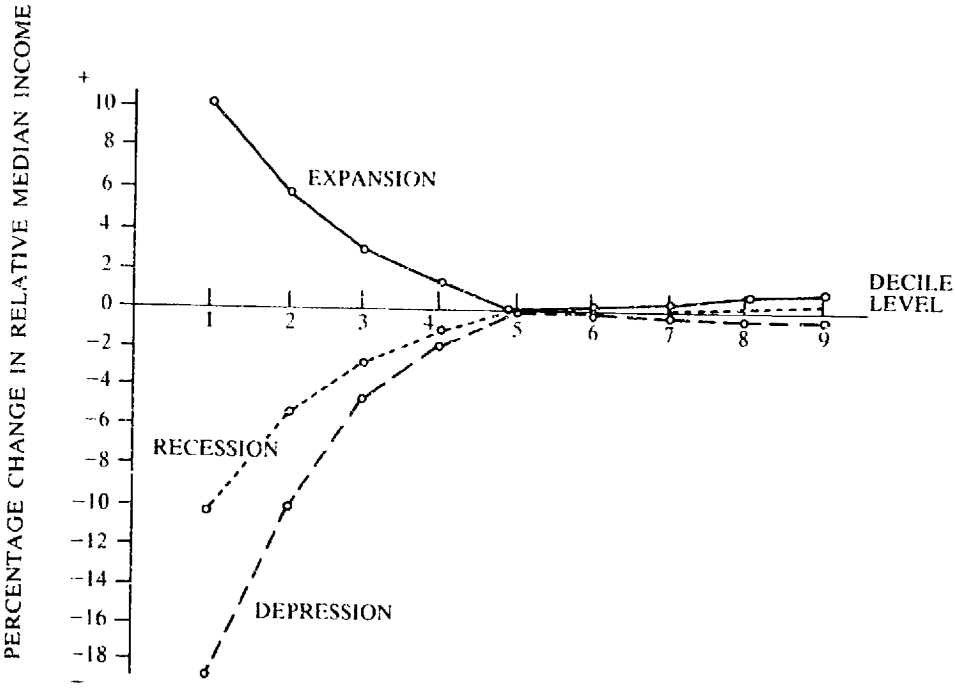
In addition, one would also expect unemployment benefits to vary directly with the unemployment rate. This has in fact been incorporated in the present model of quantile behaviour by tying unemployment benefits per unemployed person linearly to *per capita* wage income to reflect institutional arrangements concerning benefit levels. Thus with benefits per person determined by wage levels, *total* unemployment benefits do vary in proportion with the unemployment rate.

Combining Bowen and Finegan's estimates of the discouraged worker effects with Wachter's estimates of permanent and transitory wage effects on participation rates and the unemployment benefits effect of the present model,[17] one can simulate economic changes in illustrative fashion by adjusting only the aggregate wage level and overall male unemployment rate. Accordingly, three cyclical swings have been simulated for this paper. In the first, representing an upswing in economic activity, the unemployment rate has been assumed to decline from 6.5 to 3.0 percent and *per capita* wage income to increase at a rate of ten percent (five points real and five points inflation) from its mean over the 1947–70 period. In the second simulation, a recession has been characterized by a rise in the unemployment rate back up from 3.0 to 6.5 percent and a wage increase at a rate of only one percent (with an inflation rate of two percent). In the third simulation representing a rather severe recession, the unemployment rate has been assumed to rise from 3.0 to 9.0 percent and wage income (both nominal and real) to decline at a rate of two percent. Estimates of implied relative median income ratios were obtained at

[17] For details on the simulation procedure, see Beach [2], pp. 28–30.

the beginning and end of each simulation, and the changes were expressed as percentages of the average ratio values over the 1947–70 period so one could see the relative magnitudes of the resulting distributional impacts. These figures are present in Table 4.

As is again evident from these results, there appears to be a definite combined pattern of cyclical fluctuation in income concentration particularly at the lower ends of the distribution for prime aged males. This is illustrated in Figure 2 below



PERCENTAGE CHANGE IN RELATIVE MEDIAN INCOME FOR GROUP 25–34

Figure 2

for the group aged twenty-five to thirty-four. While the upper decile levels change relative to the median by less than two percent, the bottom deciles fluctuate relatively by ten to twenty percentage points.

And again, similar to the partial results noted earlier, the combined inequality behaviour for the youngest and oldest age groups differs substantially from that of the prime age groups. While the lower portions of the relative median income curves for the groups aged twenty to sixty-four vary procyclically (rising during expansion and falling during recessions), the corresponding portions of the curves for the youngest and oldest age groups appear to move anticyclically, while the upper portions fluctuate procyclically—thus resulting in the upper decile groups gaining most during expansions and losing most during recessions. Among secondary aged males fourteen to nineteen, the employment and participation rate changes turn out to be the dominating factors. Among the group sixty-five and over, however, wage income and participation rate changes are the dominating factors, particularly the latter via strong and reinforcing discouraged worker and transitory wage income effects. In sum, then, the combined or total cyclical effects may indeed be fairly substantial over particular disaggregated regions of the personal income distribution.

TABLE 4
ESTIMATED PERCENTAGE CHANGE IN RELATIVE MEDIAN INCOME OVER SIMULATED CYCLICAL CHANGES

(a) Simulated Expansion

age: \ deciles:	1	2	3	4	5	6	7	8	9
14–19	−6.60	−5.82	−5.56	−5.44	0	5.10	4.10	7.18	14.04
20–24	−1.98	−0.359	−1.03	0.096	0	−0.476	−1.35	−1.58	−1.11
25–34	10.09	5.90	2.99	1.51	0	0.126	0.334	1.27	1.32
35–44	10.47	4.44	1.66	−0.001	0	0.761	1.12	1.13	1.91
45–54	14.57	6.72	3.41	1.40	0	−3.47	0.308	0.585	1.37
55–64	8.67	4.63	3.75	2.07	0	−0.653	−0.471	0.336	0.277
65+	−5.11	−5.12	−2.62	−1.43	0	2.64	1.41	5.83	6.01

(b) Simulated Recession

age: \ deciles:	1	2	3	4	5	6	7	8	9
14–19	3.85	2.63	2.19	1.99	0	−4.05	−4.95	−6.28	−13.26
20–24	−11.71	−9.38	−4.01	−0.777	0	1.11	2.19	2.73	2.43
25–34	−10.49	−5.64	−3.13	−1.63	0	0.014	0.037	0.142	0.148
35–44	−9.30	−3.62	−1.35	0	0	0.085	0.126	0.127	0.214
45–54	−13.01	−5.43	−1.98	−1.21	0	0.558	0.632	0.661	0.750
55–64	−4.55	−2.69	−2.71	−1.40	0	−0.074	−0.053	0.038	0.031
65+	1.60	1.61	0.786	−0.477	0	−1.03	0.198	−0.641	−0.047

(c) Simulated Depression

age: \ deciles:	1	2	3	4	5	6	7	8	9
14–19	7.53	5.39	4.61	4.26	0	−7.77	−9.43	−12.36	−24.79
20–24	−15.68	−13.06	−5.35	−1.20	0	1.80	3.47	4.34	3.69
25–34	−18.54	−10.11	−5.41	−2.70	0	−0.040	−0.105	−0.399	−0.414
35–44	−17.14	−6.75	−2.44	0	0	−0.240	−0.355	−0.356	−0.602
45–54	−24.30	−10.43	−4.25	−2.29	0	0.760	0.549	0.458	0.208
55–64	−10.02	−5.62	−5.23	−2.68	0	0.209	0.151	−0.108	−0.089
65+	3.94	3.95	1.98	−0.101	0	−2.29	−0.259	−2.95	−2.25

VII. EXTENSIONS OF THE MODEL

This paper has attempted to provide a useful and novel framework for studying in detail the distributional aspects of short-run macroeconomic fluctuations. The indirect quantile approach that is forwarded involves first explaining the cyclical behaviour of a set of quantile income levels of a distribution by means of regression analysis, and then deriving the implied behaviour of various income inequality measures from the quantile equations. Thus, while income quantiles are the primary subjects of regression analysis, it is the resultant systematic fluctuations in the various inequality measures which are the basic objects of investigation.

The approach has been illustrated in this paper with a disaggregation by age and deciles for male income recipients, although various other disaggregations such as by sex, race, or occupational groups, or by finer quantile breakdowns could alternatively have been used. From a technical point of view, it would also be desirable to loosen some of the rigid parameter constraints employed in the model. The parameters of the response function in (3) are likely to vary over time, so that perhaps a more flexible approach could be based on a random coefficients procedure for seemingly unrelated equations such as recently suggested by Singh and Ullah [23]. The adding-up constraints that are actually implemented are also only approximations to the true constraints in (7), so that they could perhaps be reformulated stochastically and handled by a Theil–Goldberger [25] mixed estimation procedure.

From a macroeconomic point of view, a more extensive simulation analysis based on a more elaborate macro model would allow one to trace a wage–unemployment rate trade-off, so that one could explicitly evaluate some of the distributional costs of movements along such a trade-off. One could also obtain estimates of some of the distributional costs of inflation by simulating the effects of allowing different sources of income to grow at different rates, and by deflating different quantiles by different price indices. One could thus incorporate all together price effects, differential income effects, and unemployment rate effects of inflation upon the distribution of income. In addition, one could use the estimated quantile equations to interpret and evaluate some of the factors behind short-run fluctuations in income concentration that have occurred in the recent past, particularly in the recessions since World War II, and to investigate whether there has been any shifting in the underlying structure of income inequality associated with possible shifting of the Phillips curve in recent years. Finally, from a theoretical point of view, it would be useful "to close" our model of distributional behaviour and macro fluctuations by linking up the work in this paper with suggestions by Blinder [3] for analyzing the feedback of fluctuations in the size distribution of income upon aggregate output itself via a set of distributional consumption and wage functions. One could then analyze the interactions between output generation and income distribution in a more fully consistent framework.

APPENDIX I

Implementation of the adding-up constraints

In order to implement the adding-up restrictions in equation (7) in the text, the left-hand side of (7) is replaced by a weighted average of the estimated income deciles.

(A1)
$$(0.10) \sum_{j=1}^{9} \left(\frac{\partial y(j)}{\partial W}\right).$$

Since mean income has not been estimated in this study, the right-hand side of equation (7) is replaced by the derivative of a linear combination of the two deciles adjacent to the estimated mean, with an adjustment for the weights in (A1) summing to only 0.9. The approximation to (7) that is implemented is assumed also to hold only at the means so that the independent variables appearing in the constraint itself are evaluated at their means over the period of observation. Consequently, the implemented constraints can be expressed as general linear nonhomogeneous restrictions on the beta coefficients of the model.

APPENDIX II

Constrained parks estimation procedure

In the regression model estimated in Section IV of the paper. it is assumed that there is a set of nine "seemingly unrelated" equations, written jointly as

$$y = X\beta + u, \tag{A2}$$

where the coefficients are subject to a set of adding-up constraints,

$$R\beta = r. \tag{A3}$$

and where the disturbances for each equation are assumed to be contemporaneously correlated as well as serially dependent. In particular, the disturbances for the i'th equation are assumed to follow a first-order Markov autoregressive process,

$$u(i, t) = \rho(i)u(i, t-1) + \varepsilon(i, t)$$

where $\varepsilon(i, t)$ has conventional white noise properties.

The "constrained Parks" estimation procedure then involves the three steps: (1) obtain the Zellner seemingly-unrelated estimates of the coefficients in (A2) subject to the adding-up constraints in (A3); (2) calculate the residuals from the estimates in the first step and thence compute an estimate of the autocorrelation coefficient

$$\hat{\rho}(i) = \frac{\sum_{t=2}^{n} \hat{u}(i, t)\hat{u}(i, t-1)}{\sum_{t=2}^{n} \hat{u}^2(i, t-1)};$$

and (3) transform all the variables in the i-th equation by the matrix

$$P(i) = \begin{bmatrix} -\hat{\rho}(i) & 1 & & 0 \\ & \cdot & \cdot & \\ & & \cdot & \cdot \\ 0 & & -\hat{\rho}(i) & 1 \end{bmatrix}$$

and compute constrained seemingly-unrelated estimate of the β coefficients of the transformed equations.

It can readily be shown that such estimators are consistent and asymptotically efficient under conventional assumption. Suffice it to simply sketch the outlines of such a proof. By a slight modification of the consistency argument Zellner originally presented [29] for his seemingly unrelated regressions without cross-equation constraints, it can be demonstrated that the Zellner estimates incorporating the constraints are also consistent. Consequently, with nonstochastic independent variables, the corresponding residuals $\hat{u}(i, t)$ converge in distribution to $u(i, t)$, and $\hat{\rho}(i)$ computed from these residuals converges in probability to $\rho(i)$. Thus, if the i-th equation is transformed by $P(i)$, the vector of transformed disturbances converges in distribution to a vector of intertemporally independent disturbances. We thus have a set of equations in the transformed variables which are asymptotically identical with Zellner's seemingly unrelated equations whose coefficients are again subject to the constraint in (A3). Since the constrained Zellner estimator applied now to the transformed equations is known to yield coefficient estimates which are consistent and have the same asymptotic normal distribution as the constrained Aitken estimators based on a known covariance matrix, the same properties hold for the constrained Parks estimators.

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